EFFECT OF NONISOTHERMICITY ON BOUNDARY-LAYER SEPARATION IN A VISCOUS INCOMPRESSIBLE LIQUID

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The effect of nonisothermicity on the characteristics of an incompressible boundary layer is investigated. It is assumed that the viscosity is temperature dependent.

The possibility of boundary-layer control based on variation of the coefficient of viscosity was examined in [1, 2]. More recently, the question of the effect of nonisothermicity on the characteristics of a compressible boundary layer, particularly separation, has received considerable attention. This work is reviewed in [3]. The present note is concerned with the effect of nonisothermicity on the characteristics of an incompressible boundary layer, when the viscosity depends on temperature. The investigation is based on a numerical finite-difference solution (see [4]).

Reduced to dimensionless form, the equations of the steady-state boundary layer are written as follows:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right),$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$\Pr\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial^2 T}{\partial y^2}+D\mu\left(\frac{\partial u}{\partial y}\right)^2,$$

$$\mu=\mu(T) \tag{1}$$

with the boundary conditions

$$u=v=0,\quad T=\frac{T_0(x)-A}{B-A}\quad \text{at}\quad y=0,$$

$$u=U(x),\quad T=\frac{T_1(x)-A}{B-A}\quad \text{at}\quad y=\infty,$$

$$u=V(y),\quad T=\frac{\Theta(y)-A}{B-A}\quad \text{at}\quad x=0,$$

where $D=\mu_0 U_0^2/\rho c_p J a\Delta T$; $\Delta T = B - A$; $\Pr = \nu/a$; A and B are given constants; $U(x)$, $V(y)$, $T_0(x)$, $T_1(x)$, $\Theta(y)$ are given functions.

To integrate system (1), we use the method of finite differences [4], employing an absolutely stable implicit scheme.

Then system (1) may be approximated by the following difference system:

$$u_{i-1,k}\frac{u_{i,k}-u_{i-1,k}}{\Delta x}+v_{i-1,k}\frac{u_{i-1,k+1}-u_{i-1,k-1}}{2\Delta y}=$$

$$=\left(U\frac{\partial U}{\partial x}\right)_{i-1}+$$

$$+\frac{\mu_{i,k-\frac{1}{2}}u_{i,k-1}-\left(\mu_{i,k-\frac{1}{2}}+\mu_{i,k+\frac{1}{2}}\right)u_{i,k}+\mu_{i,k+\frac{1}{2}}u_{i,k+1}}{(\Delta y)^2};$$

$$v_{i,k}=v_{i,k-1}-\frac{\Delta y}{2\Delta x}(u_{i,k}-u_{i-1,k}+u_{i,k-1}-u_{i-1,k-1});$$

$$\Pr\left(u_{i-1,k}\frac{T_{i,k}-T_{i-1,k}}{\Delta x}+\right.$$

$$\left.+v_{i-1,k}\frac{T_{i-1,k+1}-T_{i-1,k-1}}{2\Delta y}\right)=$$

$$=\frac{T_{i,k+1}-2T_{i,k}+T_{i,k-1}}{(\Delta y)^2}+D\mu_{i-1,k}\frac{(u_{i-1,k+1}-u_{i-1,k-1})^2}{4(\Delta y)^2};$$

$$\mu_{i,k+\frac{1}{2}}=\frac{\mu_{i,k}+\mu_{i,k+1}}{2};\quad \mu_{i,k-\frac{1}{2}}=\frac{\mu_{i,k-1}+\mu_{i,k}}{2} \tag{2}$$

with the corresponding boundary conditions: $x_i = i\Delta x$, $y_k = k\Delta y$, $i = 1, 2, 3, \ldots, k = 1, 2, 3, \ldots, K$.

If the values of $u_{i-1,k}$, $v_{i-1,k}$, $T_{i-1,k}$ are known for some i, the third equation of system (2) reduces to the form

$$a_k T_{i,k-1}-2b_k T_{i,k}+c_k T_{i,k+1}=g_k, \tag{3}$$

where a_k, b_k, c_k, g_k are known quantities. We solve (3) by the pivotal method [5]. Having solved system (3), we determine $T_{i,k}$. After determining $T_{i,k}$ we deal similarly with the first equation of system (2).

After having found $u_{i,k}$ we determine $v_{i,k}$ from the second equation of system (2). We then proceed to determine $T_{i+1,k}$, $u_{i+1,k}$, $v_{i+1,k}$, etc.

The calculations were made for the case in which $U(x) = 1 - x$, $V(y) = 1$, $T_0(x) = A$, $T_1(x) = \Theta(y) = B$. Instead of the condition $u = U(x)$ at $y = \infty$ we took the condition $\partial u/\partial y = 0$ at $y_k = K\Delta y$, the point K being so selected that the condition $u_{i,k} = U(x_i)$ was satisfied with given accuracy.

It was assumed that the viscosity depends on temperature according to the Bachinskii formula $\mu(T) = \mu_0/(b_1 + b_2 T)$. All the calculations were made for lubricating oil, whose characteristics were taken from [6].

The following cases were considered:

1) viscosity independent of temperature: $\mu(T) = 1$;

2) viscosity dependent on temperature: a) wall temperature 20° C, freestream temperature 40° C. For this case

$$\mu(T) = \frac{1}{1+2.24T}, \quad A = 20°, \; B = 40°;$$

b) wall temperature 40° C, freestream temperature 20° C. For this case,

$$\mu(T) = \frac{1}{1-0.69T}, \quad A = 40°, \; B = 20°.$$

The location of the separation point was determined from the condition $\partial u/\partial y|_{y=0} = 0$, which in finite-difference form becomes

$$\frac{-3u_{i,0} + 4u_{i,1} - u_{i,2}}{2\Delta y} = 0.$$

Calculations gave the following location of the separation points:

$$0.1250 > x_{sep} > 0.1225 \text{ for } \mu = 1;$$

$$0.09750 > x_{sep} > 0.09625 \text{ for } \mu = \frac{1}{1+2.24T};$$

$$0.16750 > x_{sep} > 0.16625 \text{ for } \mu = \frac{1}{1-0.69T}.$$

Clearly, when the viscosity is temperature-dependent, the nonisothermicity has an important influence on the location of the separation points. In particular, when the wall is heated the separation point is shifted considerably downstream.

Figure 1 shows the velocity profiles at the point x = 0.05, while Fig. 2 shows the profiles near the separation points. It is clear from Fig. 1 that when the wall temperature is lower than the freestream temperature the velocity profile has a point of inflection. This leads to earlier separation. When the wall temperature is lower [sic] than the freestream temperature, a point of inflection appears only near the separation point (Fig. 2). Figure 3 shows how the nonisothermicity of the flow affects the displacement thickness $\delta^* = \frac{1}{U}\int_0^\infty (U-u)\,dx$. The integral was evaluated numerically from y_0 to $y_k = K\Delta y$ according to the trapezoidal rule. Clearly, cooling the wall reduces and heating it increases the thickness of the boundary layer.

In the computations the steps Δx, Δy, and K were so chosen that within the accuracy selected a decrease in Δx and Δy and an increase in K had no effect on the results. We finally selected $\Delta x = 0.0003125$, $\Delta y = 0.02$, K = 201 for the cases $\mu(T) = 1$ and $\mu(T) = 1/(1 + 2.24T)$, K = 301 for the case $\mu(T) = 1/1 - 0.69T$.

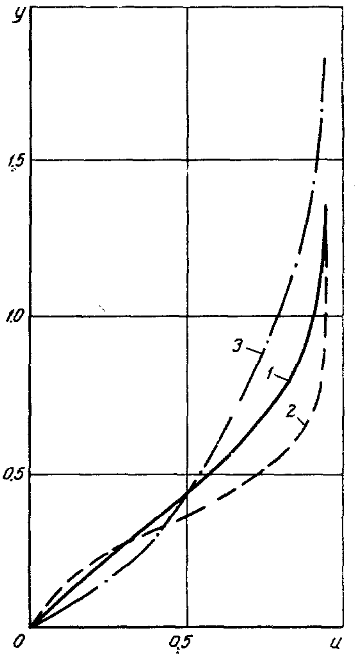

Fig. 1. Velocity profiles at x = 0.05: 1) μ = 1; 2) 1/1 + 2.24T; 3) 1/1 − 0.69T.

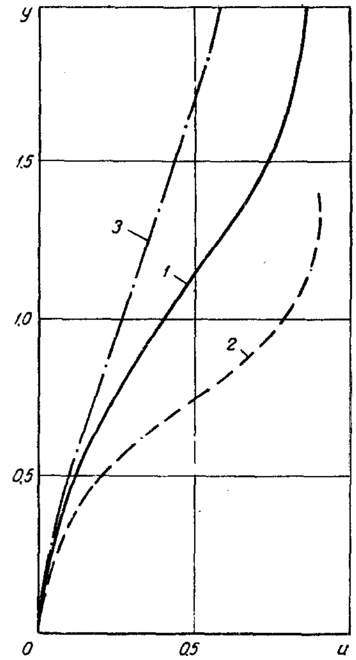

Fig. 2. Velocity profiles near separation points: 1) μ = 1 at x = 0.1225; 2) 1/1 + 2.24T at x = 0.09625; 3) 1/1 − 0.69T at x = 0.16625.

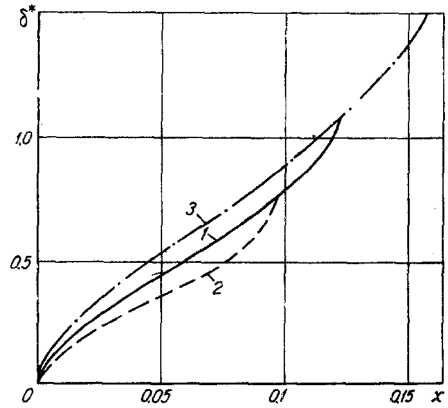

Fig. 3. Boundary-layer displacement thickness: 1) μ = 1; 2) 1/1 + 2.24T; 3) 1/1 − 0.69T.

In the calculations whose results are presented above we took Pr = 7 and D = 0 in view of the smallness of D at the selected values of the parameters.

The calculations were performed on a Minsk-2 computer.

NOTATION

u and v are velocity components; T is the temperature; U is the velocity of the potential flow; μ is the coefficient of viscosity; ρ is the density of the fluid; J is the mechanical equivalent of heat; a is the thermal diffusivity; ν is the kinematic coefficient of viscosity; U_0, μ_0, l, A, B are characteristic constants; δ^* is the displacement thickness of the boundary layer; $Re = U_0 l/\nu$; $Pr = \nu/a$; $D = \mu_0 U_0^2/\rho C_p J a \Delta T$; $\Delta T = B - A$; c_p is the specific heat at constant pressure.

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